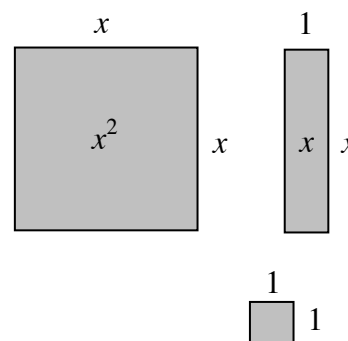
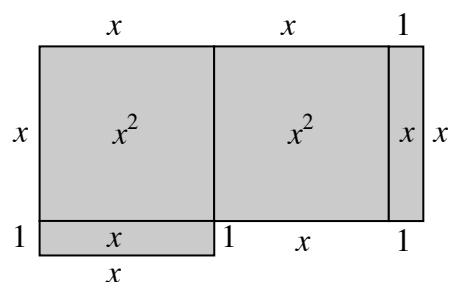


Algebraic expressions can be represented by the perimeters of algebra tiles (rectangles and squares) and combinations of algebra tiles. The dimensions of each tile are shown along its sides and the tile is named by its area as shown on the tile itself in the figures at right. When using the tiles, perimeter is the distance around the exterior of a figure.

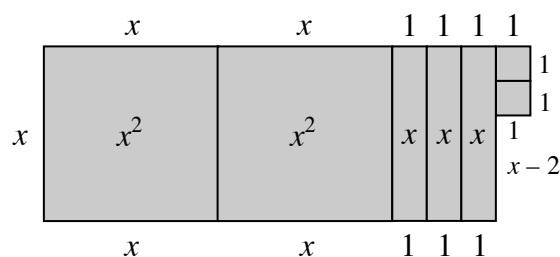


Example 1



$$P = 6x + 4 \text{ units}$$

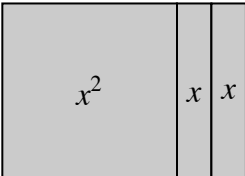
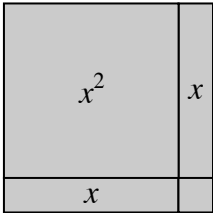
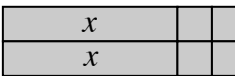
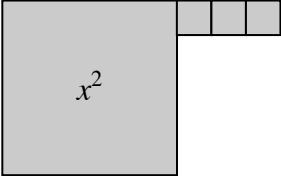
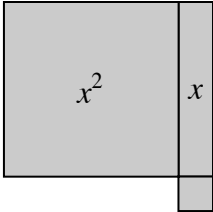
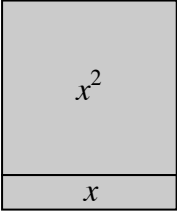
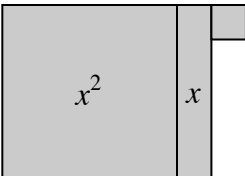
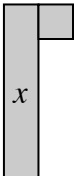
Example 2



$$P = 6x + 8 \text{ units}$$

Problems

Determine the perimeter of each figure.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 

Answers

- | | | | |
|-----------------|-----------------|-----------------|-----------------|
| 1. $4x + 4$ un. | 2. $4x + 4$ un. | 3. $2x + 8$ un. | 4. $4x + 6$ un. |
| 5. $4x + 4$ un. | 6. $4x + 2$ un. | 7. $4x + 4$ un. | 8. $2x + 4$ un. |

Algebraic expressions can also be simplified by combining (adding or subtracting) terms that have the same variable(s) raised to the same powers, into one term. The skill of combining like terms is necessary for solving equations. For additional information, see the Math Notes box in Lesson 2.1.3 of the *Core Connections, Course 3* text.

Example 1

Combine like terms to simplify the expression $3x + 5x + 7x$.

All these terms have x as the variable, so they are combined into one term, $15x$.

Example 2

Simplify the expression $3x + 12 + 7x + 5$.

The terms with x can be combined. The terms without variables (the constants) can also be combined.

$$3x + 12 + 7x + 5$$

$$3x + 7x + 12 + 5$$

$$10x + 17$$

Note that in the simplified form the term with the variable is listed before the constant term.

Example 3

Simplify the expression $5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$.

$$5x + 4x^2 + 10 + 2x^2 + 2x - 6 + x - 1$$

$$4x^2 + 2x^2 + 5x + 2x + x + 10 - 6 - 1$$

$$6x^2 + 8x + 3$$

Note that terms with the same variable but with different exponents are not combined and are listed in order of decreasing power of the variable, in simplified form, with the constant term last.

Example 4

The algebra tiles, as shown in the *Algebra Tiles and Perimeter* section, are used to model how to combine like terms.

The large square represents x^2 , the rectangle represents x , and the small square represents one. We can only combine tiles that are alike: large squares with large squares, rectangles with rectangles, and small squares with small squares. If we want to combine: $2x^2 + 3x + 4$ and $3x^2 + 5x + 7$, visualize the tiles to help combine the like terms:

$$2x^2 \text{ (2 large squares)} + 3x \text{ (3 rectangles)} + 4 \text{ (4 small squares)} \\ + 3x^2 \text{ (3 large squares)} + 5x \text{ (5 rectangles)} + 7 \text{ (7 small squares)}$$

The combination of the two sets of tiles, written algebraically, is: $5x^2 + 8x + 11$.

Example 5

Sometimes it is helpful to take an expression that is written horizontally, circle the terms with their signs, and rewrite *like* terms in vertical columns before you combine them:

$$\begin{array}{r} (2x^2 - 5x + 6) + (3x^2 + 4x - 9) \\ \textcircled{2x^2} - \textcircled{5x} + \textcircled{6} + \textcircled{3x^2} + \textcircled{4x} - \textcircled{9} \\ \begin{array}{r} 2x^2 \quad -5x \quad +6 \\ + 3x^2 \quad +4x \quad -9 \\ \hline 5x^2 \quad -x \quad -3 \end{array} \end{array}$$

This procedure may make it easier to identify the terms as well as the sign of each term.

Problems

Combine the following sets of terms.

- $(2x^2 + 6x + 10) + (4x^2 + 2x + 3)$
- $(3x^2 + x + 4) + (x^2 + 4x + 7)$
- $(8x^2 + 3) + (4x^2 + 5x + 4)$
- $(4x^2 + 6x + 5) - (3x^2 + 2x + 4)$
- $(4x^2 - 7x + 3) + (2x^2 - 2x - 5)$
- $(3x^2 - 7x) - (x^2 + 3x - 9)$
- $(5x + 6) + (-5x^2 + 6x - 2)$
- $2x^2 + 3x + x^2 + 4x - 3x^2 + 2$
- $3c^2 + 4c + 7x - 12 + (-4c^2) + 9 - 6x$
- $2a^2 + 3a^3 - 4a^2 + 6a + 12 - 4a + 2$

Answers

- $6x^2 + 8x + 13$
- $4x^2 + 5x + 11$
- $12x^2 + 5x + 7$
- $x^2 + 4x + 1$
- $6x^2 - 9x - 2$
- $2x^2 - 10x + 9$
- $-5x^2 + 11x + 4$
- $7x + 2$
- $-c^2 + 4c + x - 3$
- $3a^3 - 2a^2 + 2a + 14$

SIMPLIFYING EXPRESSIONS (ON AN EXPRESSION MAT) 2.1.3 – 2.1.5

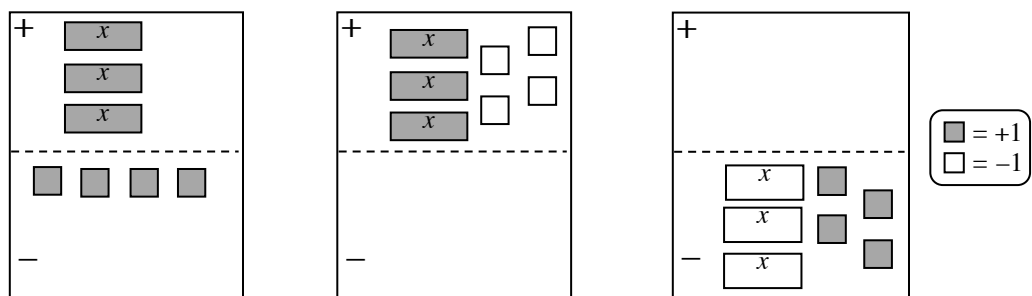
Two Region Expression Mats

An Expression Mat is an organizational tool that is used to represent algebraic expressions. Pairs of Expression Mats can be modified to make an Equation Mat. The upper half of an Expression Mat is the positive region and the lower half is the negative region. Positive algebra tiles are shaded and negative tiles are blank. A matching pair of tiles with one tile shaded and the other one blank represents zero (0).

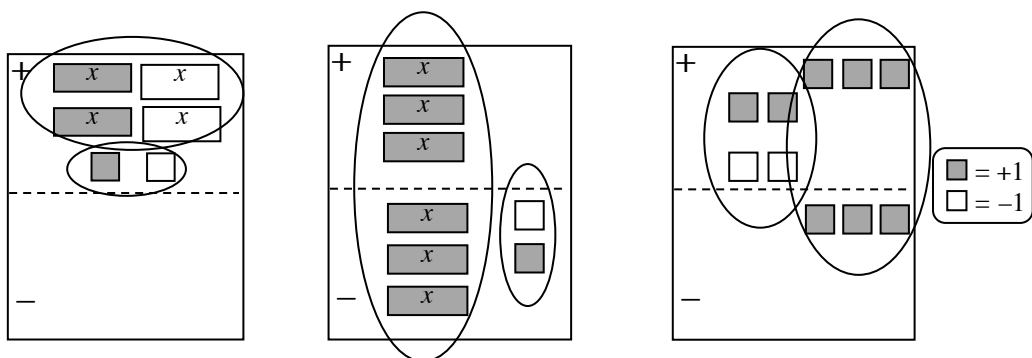
Tiles may be removed from or moved on an expression mat in one of three ways: (1) removing the same number of opposite tiles in the same region; (2) flipping a tile from one region to another. Such moves create “opposites” of the original tile, so a shaded tile becomes un-shaded and an un-shaded tile becomes shaded; and (3) removing an equal number of identical tiles from both the “+” and “-” regions. See the Math Notes box in Lesson 2.1.6 of the *Core Connections, Course 3* text.

Examples

$3x - 4$ can be represented various ways.



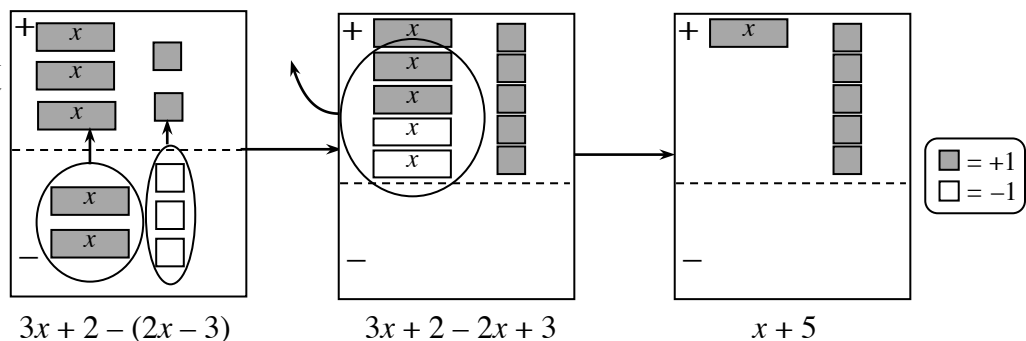
The Expression Mats at right all represent zero.



Example 1

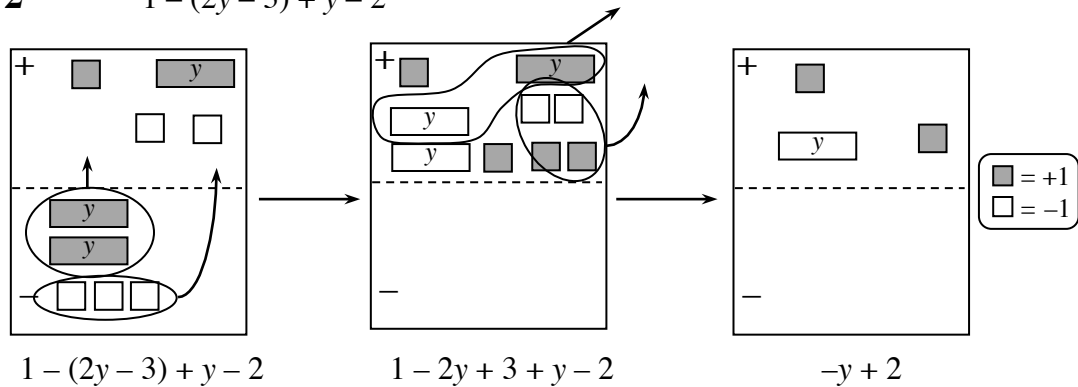
$$3x + 2 - (2x - 3)$$

Expressions can be simplified by moving tiles to the top (change the sign) and looking for zeros.



Example 2

$$1 - (2y - 3) + y - 2$$



Problems

Simplify each expression.

1.

2.

3.

4.

5.

6.

7. $3 + 5x - 4 - 7x$

8. $-x - 4x - 7$

9. $-(-x + 3)$

10. $4x - (x + 2)$

11. $5x - (-3x + 2)$

12. $x - 5 - (2 - x)$

13. $1 - 2y - 2y$

14. $-3x + 5 + 5x - 1$

15. $3 - (y + 5)$

16. $-(x + y) + 4x + 2y$

17. $3x - 7 - (3x - 7)$

18. $-(x + 2y + 3) - 3x + y$

Answers

- | | | | | | |
|-----|-----------|-----|-----------|-----|---------------|
| 1. | 0 | 2. | $2x + 2$ | 3. | $2y + 2$ |
| 4. | $-5x + 2$ | 5. | $2y - 1$ | 6. | $-y + 5$ |
| 7. | $-2x - 1$ | 8. | $-5x - 7$ | 9. | $x - 3$ |
| 10. | $3x - 2$ | 11. | $8x - 2$ | 12. | $2x - 7$ |
| 13. | $-4y + 1$ | 14. | $2x + 4$ | 15. | $-y - 2$ |
| 16. | $3x + y$ | 17. | 0 | 18. | $-4x - y - 3$ |

COMPARING QUANTITIES (ON AN EXPRESSION MAT) 2.1.5 – 2.1.8

Combining two Expression Mats into an Expression Comparison Mat creates a concrete model for simplifying (and later solving) inequalities and equations.

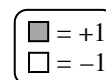
Tiles may be removed or moved on the mat in the following ways:

- (1) Removing the same number of opposite tiles (zeros) on the same side;
- (2) Removing an equal number of identical tiles (balanced set) from both the left and right sides;
- (3) Adding the same number of opposite tiles (zeros) on the same side; and
- (4) Adding an equal number of identical tiles (balanced set) to both the left and right sides.

These strategies are called “legal moves.”

After moving and simplifying the Expression Comparison Mat, students are asked to tell which side is greater. Sometimes it is only possible to tell which side is greater if you know possible values of the variable.

Example 1



Determine which side is greater by using legal moves to simplify.

Step 1: Remove balanced set

Left: Right:

Which is greater?

Step 2: Remove zeros

Left:

Right:

Which is greater?

Step 3: Remove balanced set

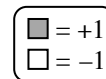
Left:

Right:

Which is greater?

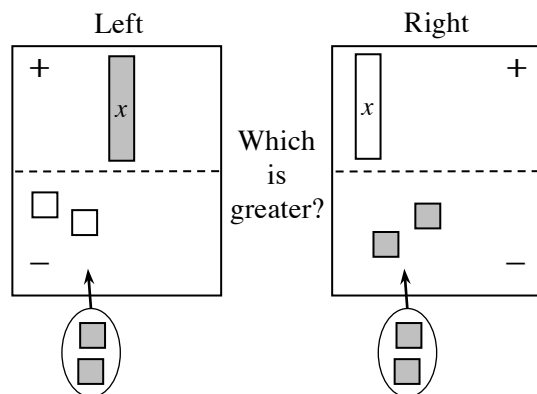
The left side is greater because after Step 3: $4 > 0$. Also, after Step 2: $6 > 2$. Note that this example shows only one of several possible strategies.

Example 2

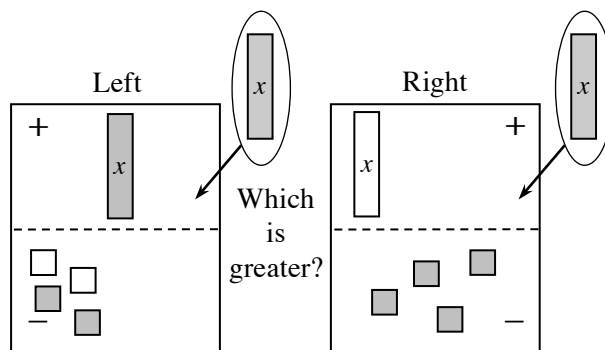


Use legal moves so that all the x -variables are on one side and all the unit tiles are on the other.

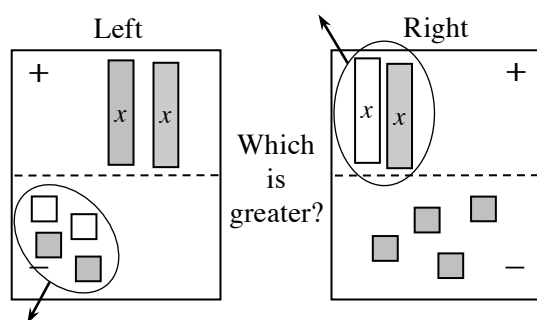
Step 1: Add balanced set



Step 2: Add balanced set



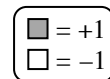
Step 3: Remove zeros



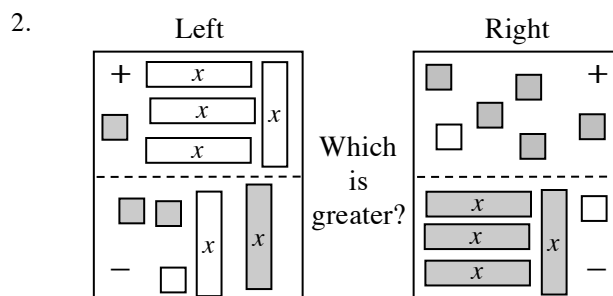
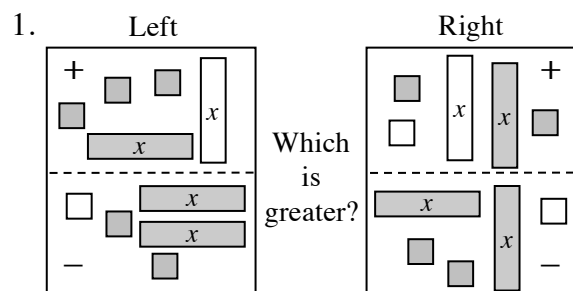
What remains is $2x$ on Mat A and -4 on Mat B. There are other possible arrangements. Whatever the arrangement, it is not possible to tell which side is greater because we do not know the value of " x ." Students are expected to record the results algebraically as directed by the teacher. One possible recording is shown below.

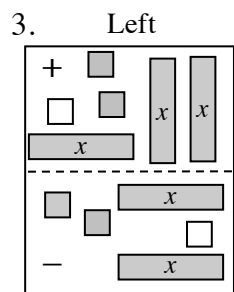
Left	Right
$x - (-2)$	$-x - (2)$
$x - (-2 + 2)$	$-x - (2 + 2)$
$x + x - (-2 + 2)$	$-x + x - (2 + 2)$
$2x$	4

Problems

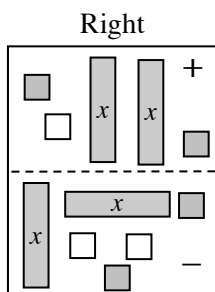


For each of the problems below, use the strategies of removing zeros or simplifying by removing balanced sets to determine which side is greater, if possible. Record your steps.

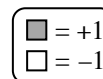




Which
is
greater?



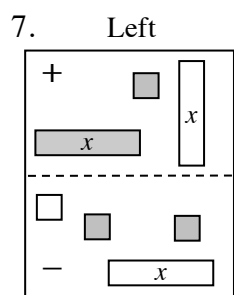
4. Left: $5 + (-8)$
Right: $-7 + 6$



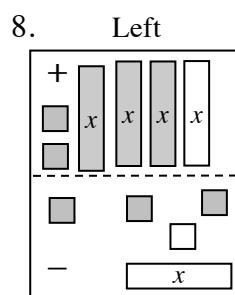
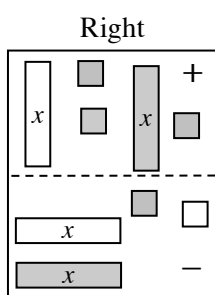
5. Left: $2(x + 3) - 2$
Right: $4x - 2 - x + 4$

6. Left: $4 + (-2x) + 4x$
Right: $x^2 + 2x + 3 - x^2$

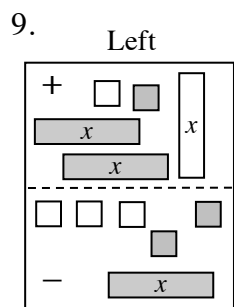
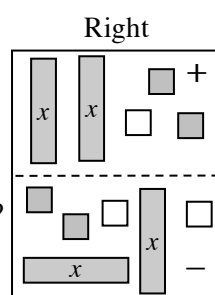
For each of the problems below, use the strategies of removing zeros or adding/removing balanced sets so that all the x -variables are on one side and the unit tiles are on the other. Record your steps.



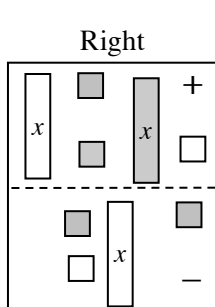
Which
is
greater?



Which
is
greater?



Which
is
greater?



10. Left: $3x - 2$
Right: $2x + 1$

11. Left: $4x + 2 + (-5)$
Right: $2x + 3 + (-8)$

12. Left: $2x + 3 + (-8)$
Right: $-x - 3$

Answers (Answers to problems 7 through 12 may vary.)

- | | | |
|-------------------|-------------------------|-------------------------|
| 1. left | 2. right | 3. not possible to tell |
| 4. right | 5. not possible to tell | 6. left |
| 7. A: x ; B: 3 | 8. A: $3x$; B: 1 | 9. A: 1; B: x |
| 10. A: x ; B: 3 | 11. A: $2x$; B: -2 | 12. A: $3x$; B: -6 |

Using a Four-Region Equation Mat

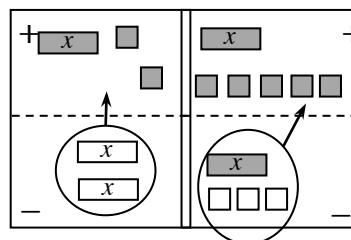
Combining two Expression Mats into an Equation Mat creates a concrete model for solving equations. Practice solving equations using the model will help students transition to solving equations abstractly with better accuracy and understanding.

In general, and as shown in the first example below, the negative in front of the parenthesis causes everything inside to “flip” from the top to the bottom or the bottom to the top of an Expression Mat, that is, all terms in the expression change signs. After simplifying the parentheses, simplify each Expression Mat. Next, isolate the variables on one side of the Equation Mat and the non-variables on the other side by removing matching tiles from both sides. Then determine the value of the variable. Students should be able to explain their steps. See the Math Notes boxes in Lessons 2.1.9 and 3.2.3 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 3* Checkpoint 5 materials.

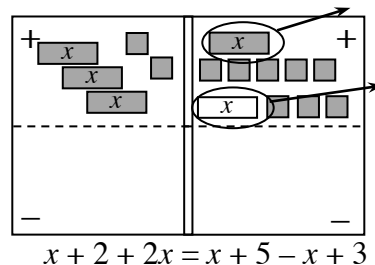
Procedure and Example

Solve $x + 2 - (-2x) = x + 5 - (x - 3)$.

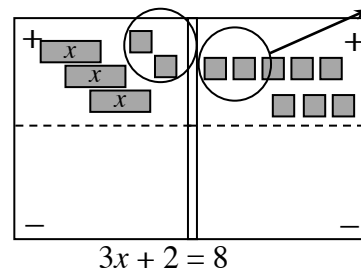
First build the equation on the Equation Mat.



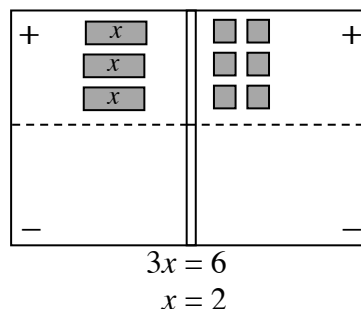
Second, simplify each side using legal moves on each Expression Mat, that is, on each side of the Equation Mat.



Isolate x -terms on one side and non- x -terms on the other by removing matching tiles from both sides of the Equation Mat.



Finally, since both sides of the equation are equal, determine the value of x .



Once students understand how to solve equations using an Equation Mat, they may use the visual experience of moving tiles to solve equations with variables and numbers. The procedures for moving variables and numbers in the solving process follow the same rules.

Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, $2 = 4$), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 2 = x + 2$) it means that *all numbers* are solutions. See the Math Notes box in Lesson 3.2.4 of the *Core Connections, Course 3* text.

Example 1 Solve: $3x + 3x - 1 = 4x + 9$

Solution:	$3x + 3x - 1 = 4x + 9$	problem
	$6x - 1 = 4x + 9$	simplify
	$2x = 10$	add 1, subtract $4x$ on each side
	$x = 5$	divide

Example 2 Solve: $-2x + 1 - (-3x + 3) = -4 + (-x - 2)$

Solution:	$-2x + 1 - (-3x + 3) = -4 + (-x - 2)$	problem
	$-2x + 1 + 3x - 3 = -4 - x - 2$	remove parentheses (flip)
	$x - 2 = -x - 6$	simplify
	$2x = -4$	add x , add 2 to each side
	$x = -2$	divide

Problems

Solve each equation.

- | | |
|-------------------------------------|--------------------------------------|
| 1. $2x - 3 = -x + 3$ | 2. $1 + 3x - x = x - 4 + 2x$ |
| 3. $4 - 3x = 2x - 6$ | 4. $3 + 3x - (x - 2) = 3x + 4$ |
| 5. $-(x + 3) = 2x - 6$ | 6. $-4 + 3x - 1 = 2x + 1 + 2x$ |
| 7. $-x + 3 = 10$ | 8. $5x - 3 + 2x = x + 7 + 6x$ |
| 9. $4y - 8 - 2y = 4$ | 10. $9 - (1 - 3y) = 4 + y - (3 - y)$ |
| 11. $2x - 7 = -x - 1$ | 12. $-2 - 3x = x - 2 - 4x$ |
| 13. $-3x + 7 = x - 1$ | 14. $1 + 2x - 4 = -3 - (-x)$ |
| 15. $2x - 1 - 1 = x - 3 - (-5 + x)$ | 16. $-4x - 3 = x - 1 - 5x$ |
| 17. $10 = x + 6 + 2x$ | 18. $-(x - 2) = x - 5 - 3x$ |
| 19. $6 - x - 3 = 4x - 8$ | 20. $0.5x - (-x + 3) = x - 5$ |

Answers

- | | | | | |
|--------------------|------------------------|----------------|------------------------|--------------|
| 1. $x = 2$ | 2. $x = 5$ | 3. $x = 2$ | 4. $x = 1$ | 5. $x = 1$ |
| 6. $x = -6$ | 7. $x = -7$ | 8. no solution | 9. $x = 6$ | 10. $x = -7$ |
| 11. $x = 2$ | 12. all
numbers | 13. $x = 2$ | 14. $x = 0$ | 15. $x = 2$ |
| 16. no
solution | 17. $x = 1\frac{1}{3}$ | 18. $x = -7$ | 19. $x = 2\frac{1}{5}$ | 20. $x = -4$ |